Graphs, 3rd test (2nd try)

January 15th, 2009

Exercise 1. Does there exist an $n \in \mathbb{N}$, such that there exist simple planar bipartite graphs G_1 and G_2 with n vertices and $2n - 4$ edges, such that $G_1 \not\cong G_2$?

Exercise 2. For any odd $n \in \mathbb{N}$, define the simple graphs G_n as follows:

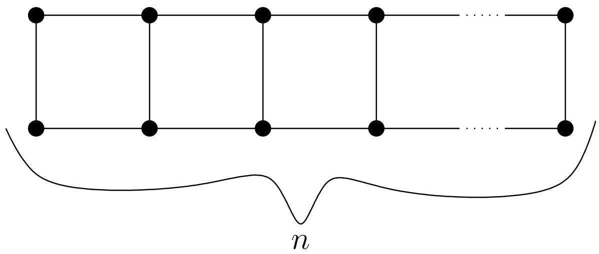
- the set of vertices V_n of G_n is $\{1, 3, 5, \dots, n\}$;
- two numbers $x, y \in V_n$ are connected with an edge iff $\gcd(x, y) > 1$.

For which values of n is G_n planar?

Exercise 3. The Ramsey number $r(k, l)$ is defined as the smallest n, such that for any coloring of the edges of K_n with two colors, there exists a monochromatic copy of K_k of the first color, or a monochromatic copy of K_l of the second color. We can generalize $r(k, l)$ as follows: for any two (simple) graphs G_1, G_2 let $r(G_1, G_2)$ be the smallest n, such that for any coloring of the edges of K_n with two colors, there is an $i \in \{1, 2\}$, such that G_i is a subgraph (not necessarily induced) of the graph made up of the edges of i-th color.

Show that if T is any tree with n vertices, then
$r(K_m, T) \geq (m - 1)(n - 1) + 1$.

Exercise 4. Find the chromatic polynomial of the $2n$-vertex graph G_n, depicted below.



The usage of written/printed materials is allowed.

Graafid, 3. kontrolltöö (teine katse)

15. jaanuar 2009

Ülesanne 1. Kas leidub selline $n \in \mathbb{N}$, nii et leiduvad tasandilised kahealuselised n tipu ja $(2n-4)$ servaga lihtgraafid G_1 ja G_2 nii, et $G_1 \ncong G_2$?

Ülesanne 2. Iga paaritu $n \in \mathbb{N}$ jaoks defineerime lihtgraafi G_n järgmiselt:

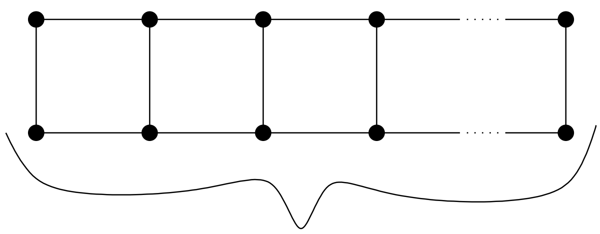
- graafi G_n tipuhulk V_n on $\{1, 3, 5, \ldots, n\}$;
- kaks arvu $x, y \in V_n$ on servaga ühendatud parajasti siis, kui $\mathrm{SÜT}(x, y) > 1$.

Milliste n väärtuste jaoks on G_n tasandiline?

Ülesanne 3. Ramsey arv $r(k, l)$ on defineeritud kui vähim selline n, et ükskõik mis viisil me ka ei värviks graafi K_n servi kahe värviga, leidub saadud graafis kas esimest värvi koopia graafist K_k või teist värvi koopia graafist K_l. Arve $r(k, l)$ võib järgmisel viisil üldistada: iga kahe lihtgraafi G_1, G_2 jaoks olgu $r(G_1, G_2)$ selline n, et ükskõik mis viisil me ka ei värviks graafi K_n servi kahe värviga, leidub selline i, et graafil, mille moodustavad ainult i-ndat värvi servad, on alamgraaf (mitte tingimata indutseeritud) G_i.

Näita, et kui T on suvaline n tipuga puu, siis $r(K_m, T) \geq (m-1)(n-1)+1$.

Ülesanne 4. Leia alloleva $2n$-tipulise graafi G_n kromaatiline polünoom.



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Paberkandjal materjale tohib kasutada.