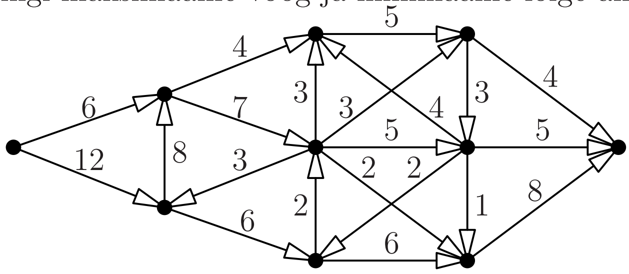
# Graafid, 2. kontrolltöö

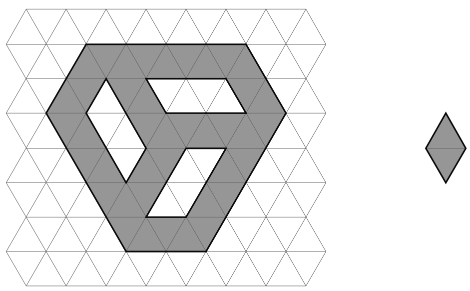
**18. aprill 2011**

**Ülesanne 1.** Leia mingi maksimaalne voog ja minimaalne lõige alltoodud võrgus.



**Ülesanne 2.** Olgu $G = (X \cup Y, E)$ kahealuseline graaf alustega $X$ ja $Y$. Olgu $d_X = \max_{v \in X} \deg(v)$ ja $X' = \{v \in X \mid \deg(v) = d_X\}$. Näita, et graafis $G$ leidub kooskõla $M \subseteq E$, nii et $\deg_M(x) = 1$ iga $x \in X'$ jaoks.

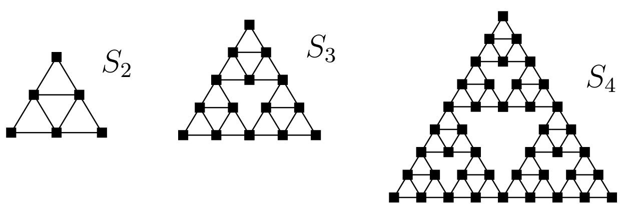
**Ülesanne 3.** Kas vasakpoolne kujund on kaetav parempoolsete kahest kolmnurgast koosnevate kujunditega (neid tohib vastavalt vajadusele pöörata)?



**Ülesanne 4.** Defineerime lihtgraafid $S_1, S_2, S_3, \ldots$, mida võiks nimetada *Sierpinski graafideks*. Graafiks $S_1$ on $K_3$. Nimetame selle graafi kolme tippu *vasakuks*, *paremaks* ja *ülemiseks*. Graafi $S_{i+1}$ saame kui graafi $S_i$ kolme eksemplari (mida nimetame samuti *vasakuks*, *paremaks* ja *ülemiseks*) ühendi, kus järgmised tipupaarid loeme üheksainsaks tipuks:

- $S_i$ vasaku eksemplari ülemine tipp ja $S_i$ ülemise eksemplari vasak tipp;
- $S_i$ parema eksemplari ülemine tipp ja $S_i$ ülemise eksemplari parem tipp;
- $S_i$ parema eksemplari vasak tipp ja $S_i$ vasaku eksemplari parem tipp.

Graafi $S_{i+1}$ tipp $X$ (kus $X \in \{\text{vasak}, \text{parem}, \text{ülemine}\}$) on $S_i$ $X$ eksemplari $X$ tipp. All on toodud mõned näited graafidest $S_i$. Mitme värviga on korrektselt värvitavad graafide $S_i$ servad?



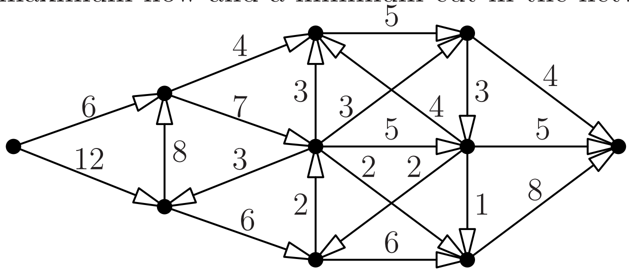

(Paberkandjal) materjale tohib kasutada.

Kõik ülesanded on võrdse kaaluga.

# Graphs, 2nd test

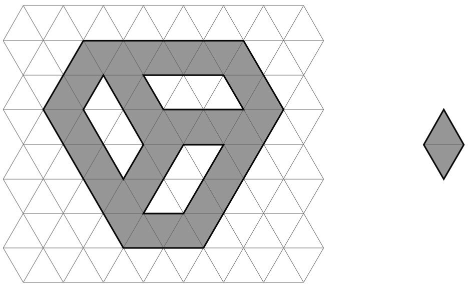
**18 April 2011**

**Exercise 1.** Find a maximum flow and a minimum cut in the network below.




**Exercise 2.** Let $G = (X \cup Y, E)$ be a bipartite graph with parts $X$ and $Y$ of the vertex set. Define $d_X = \max_{v \in X} \deg(v)$ and $X' = \{v \in X \mid \deg(v) = d_X\}$. So that $G$ has a matching $M \subseteq E$, such that $\deg_M(x) = 1$ for any $x \in X'$ jaoks.

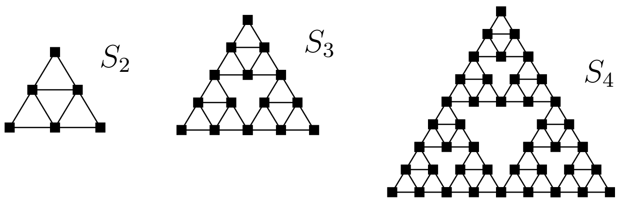
**Exercise 3.** Is it possible to cover the shape in the left with the two-triangle figures (which may be rotated as necessary) shown to the right?



**Exercise 4.** Let us define simple graphs $S_1, S_2, S_3, \ldots$ that might be called *Sierpinski graphs*. Graph $S_1$ equals $K_3$. Let us call its three vertices the *left vertex*, the *right vertex* and the *top vertex*. The graph $S_{i+1}$ is obtained as the union of three copies of $S_i$ (that we also call *left*, *right* and *top*, respectively), where we identify the following pairs of vertices:

- the top vertex of the left $S_i$ and the left vertex of the top $S_i$;
- the top vertex of the right $S_i$ and the right vertex of the top $S_i$;
- the right vertex of the left $S_i$ and the left vertex of the right $S_i$.

The $X$ vertex (where $X \in \{\text{left}, \text{right}, \text{top}\}$) of $S_{i+1}$ is the $X$ vertex of the $X$ copy of $S_i$. Some examples of the graphs $S_i$ are given below. How many colors are needed to correctly color the edges of $S_i$?




This test is open-book.

All exercises are worth the same amount of points