

Cryptographic protocols

(MTAT.07.005, 4 AP / 6 ECTS)

Lectures and Tue 12-16 hall 404

Exercises: Thu 10-12 hall 404

homepage:

http://www.ut.ee/~peeter_l/teaching/krprot07s

(contains lecture materials)

Grading: Exam in January.

(maybe I'll come up with something that can replace parts of it)

- Cryptology I was mostly about primitives.
 - (A)symmetric encryption, signatures, MACs, hash functions, etc.
- To achieve the security goals of systems, several of them have to be used together.
- This gives us protocols.
- It's quite easy to use the primitives in the wrong way.
- This makes the protocols insecure, although the primitives themselves might have been secure.

- Example: Alice and Bob want to set up a private channel between themselves.
- They know each other's public keys K_A and K_B .
- Alice generates a new key K_{AB} of some symmetric encryption system.
- Alice sends K_{AB} to B , encrypted with K_B .

$$A \longrightarrow B : \{[K_{AB}]\}_{K_B}$$

- Bob decrypts and learns K_{AB} .
- Alice and Bob use K_{AB} to encrypt messages between each other.
 - Assume it also provides integrity.

- Who sent the key to Bob?
 - Alice did. . .
 - Include Alice's name in the message:

$$A \longrightarrow B : \{[A, K_{AB}]\}_{K_B}$$

- When was it sent?
 - consider **replay attacks**.
 - The adversary may somehow know the old session keys.

- Include a **timestamp** to the message:

$$A \longrightarrow B : \{[A, T, K_{AB}]\}_{K_B}$$

- B must check that T is not far off.
- How do A and B synchronize their clocks?
- What if the attacker takes over B 's NTP server?

- Better: include a **nonce** in the message:

$$A \longrightarrow B : \{[A, N, K_{AB}]\}_{K_B}$$

- Nonce \equiv random bit-string.
- B must check that it has not received that N before.

- B has to store all N -s it receives...
 - What if his hard drive fails?
- The attacker may
 1. not deliver the message $\{[A, N, K_{AB}]\}_{K_B}$;
 2. wait until it learns K_{AB} ;
 3. deliver $\{[A, N, K_{AB}]\}_{K_B}$.

- B needs to know that A sent that message **recently**.
- B must answer to A and then A must answer to B .

$$A \longrightarrow B : \{[A, N, K_{AB}]\}_{K_B}$$

$$B \longrightarrow A : \{[???\}_{K_A}$$

$$A \longrightarrow B : \{[???\}_{K_B}$$

- 2nd and 3rd message have to mention N .

$$A \longrightarrow B : \{[A, N, K_{AB}]\}_{K_B}$$

$$B \longrightarrow A : \{[N]\}_{K_A}$$

$$A \longrightarrow B : \{[N]\}_{K_B}$$

- A must verify that it sent N recently.
- B must do the same verification after 3rd message.
- What replay possibilities are there?

- B needs a nonce, too.

$$A \longrightarrow B : \{[A, N_A, K_{AB}]\}_{K_B}$$

$$B \longrightarrow A : \{[N_A, N_B]\}_{K_A}$$

$$A \longrightarrow B : \{[N_A, N_B]\}_{K_B}$$

- Assume now that Alice wants to talk to Charlie:

$$A \longrightarrow C : \{A, N_A, K_{AC}\}_{K_C}$$

- But Charlie is bad...

$$C(A) \longrightarrow B : \{A, N_A, K_{AC}\}_{K_B}$$

- Bob responds, thinking that Alice is talking to him:

$$B \longrightarrow C(A) : \{N_A, N_B\}_{K_A}$$

- Charlie simply forwards that message:

$$C \longrightarrow A : \{N_A, N_B\}_{K_A}$$

- Alice decrypts that pair of nonces for Charlie:

$$A \longrightarrow C : \{[N_A, N_B]\}_{K_C}$$

- and Charlie can respond to Bob:

$$C(A) \longrightarrow B : \{[N_A, N_B]\}_{K_B}$$

- Now Bob thinks that it shares the key K_{AC} with Alice, but Charlie also knows that key.

- B 's answer must contain his name:

$$A \longrightarrow B : \{[A, N_A, K_{AB}]_{K_B}\}$$

$$B \longrightarrow A : \{[N_A, N_B, B]_{K_A}\}$$

$$A \longrightarrow B : \{[N_A, N_B]_{K_B}\}$$

- Is this protocol secure? Maybe...
- Are all its parts necessary?
 - Do we need all components of all messages?
 - Does everything have to be under encryption?

Probably not.

More fundamental questions?

- What is the security property?
- What did this $A \longrightarrow B : M$ actually mean? Or:
- What is the execution model?
 - What data and control structures do the parties use?
 - How are the messages relayed?
 - How are the parties scheduled?
 - Where is the adversary?
 - * How are the parties corrupted and the keys leaked?

We do not need answers to these questions as long as we are just showing attacks against protocols.

Following the formalisms in Cryptology I:

- Each party is an implementation of some interface. It has methods for
 - starting a session;
 - receiving a message and producing an answer;
 - maybe something more.
- The adversary has a method “run” that takes all participants as its arguments.
 - More generally: there is an environment with a method “run” that takes both the participants and the adversary as arguments.
 - The implementation of this environment is fixed. This defines the scheduling and the relaying of messages.

- The security property is an environment inside which the adversary (together with the participants) is run.
- In the more general case, we can augment the environment to also check whether the security property still holds.
- Then we can talk about probabilities that $b = b^*$, or that *bad* is set to true.
- We can analyse the protocols by doing transformations...or in any other way.
 - We'll definitely meet the hybrid argument somewhere in this course.

- Such analysis may be hard...
 - but we'll be rewarded with rigorous security proofs.
- But, intuitively, what are the things that an adversary may do?

- Capture messages sent by one party to another.
 - Learn the intended sender and recipient.
- Send a message it has constructed to any party.
 - ...faking the sender.
- Generate new keys, nonces, ...
- Construct new messages from the ones its has.
 - Only applying “legitimate” constructors.
 - Because everything else will be weeded out by other parties...
- Decompose tuples. Decrypt if it knows the key.

The adversary cannot do things like:

- Learn anything about M from $\{M\}_K$.
- Transform $\{M_1\}_K, \dots, \{M_n\}_K$ to $\{M'\}_K$ for M' related to M_1, \dots, M_n , not knowing the key K .
- ...or construct any $\{M\}_K$ without knowing K at all.

Hence the encryption must provide message integrity, too.

Such encryption is often called [perfect](#).

Messages can be expressed as terms (trees). Let us have

- a countable set $\mathbf{Keys}_{\text{sym}}$ of keys for symmetric encryption;
- a countable set $\mathbf{Keys}_{\text{dec}}$ of decryption keys for asymmetric encryption;
- a countable set $\mathbf{Keys}_{\text{sig}}$ of signature keys.
- a countable set \mathbf{Nonce} of nonces.

The elements of these sets do not have any internal structure besides their identity.

There are also atomic messages expressing constants and payloads.

There are the constructors:

- $\text{pk}(K)$ gives the public key corresponding to secret $K \in \mathbf{Keys}_{\text{dec}} \cup \mathbf{Keys}_{\text{sig}}$.
- (M_1, \dots, M_n) is the tuple of the messages M_1, \dots, M_n .
- $\{M\}_K$, $\llbracket M \rrbracket_{K_p}$, $\{\!\{M\}\!\}_{K_s}$ are the symmetric, asymmetric encryption and signatures.
 - If we model randomized primitives then there is the third argument, too — the random coins.
- $h(M)$ is the digest of M .

A party can apply a constructor if it knows all of its arguments.

A **party**, including the adversary, is a **process**. It can

- Construct new atomic messages from sets $\mathbf{Keys}_{\text{sym}}$, $\mathbf{Keys}_{\text{dec}}$, $\mathbf{Keys}_{\text{sig}}$, \mathbf{Nonce} .
- Apply constructors of messages.
- Send messages.
- Receive messages.
- Decompose messages:
 - Take components of tuples.
 - Decrypt, if it knows the correct key.
- Check the equality of messages.
- Verify signatures, if it knows the correct verification key.
- Check if a decomposition would succeed.

Structure of data — a set of variables that take messages as values.

The control structures:

- Sequential composition.
- if-then-else.
- Parallel composition.
 - In $P \mid Q$, processes P and Q do not share the state.
- **Replication**.
 - $!P$ — a countable number of processes P run in parallel.

Other constructs as needed...

A protocol consists of

- The initialization of common variables;
 - Mainly long-term keys
- The parallel composition of all parties.

The protocol is executed in parallel with the adversary.

Our example:

$$A \longrightarrow B : \{[A, N_A, K_{AB}]\}_{K_B}$$

$$B \longrightarrow A : \{[N_A, N_B, B]\}_{K_A}$$

$$A \longrightarrow B : \{[N_A, N_B]\}_{K_B}$$

Names \cong public keys

$$A \longrightarrow B : \{[K_A, N_A, K_{AB}]\}_{K_B}$$

$$B \longrightarrow A : \{[N_A, N_B, K_B]\}_{K_A}$$

$$A \longrightarrow B : \{[N_A, N_B]\}_{K_B}$$

$P_A(K_B)$ is

$N_A :=$ new nonce; $K_{AB} :=$ new symkey;

$m_1 := \{[\text{pk}(SK_A), N_A, K_{AB}]\}_{K_B}$; send m_1 ;

$m_2 :=$ receive;

$dm_2 \stackrel{?}{:=} \text{dec}(SK_A, m_2)$;

$(N'_A, N_B, K'_B) \stackrel{?}{:=} dm_2$;

if $N_A \neq N'_A \vee K_B \neq K'_B$ then stop;

$m_3 := \{[N_A, N_B]\}_{K_B}$; send m_3

SK_A is the decryption key of party A.

$P_B(K_A)$ is

$m_1 := \text{receive}; dm_1 \stackrel{?}{:=} \text{dec}(SK_B, m_1);$

$(K'_A, N_A, K_{AB}) \stackrel{?}{:=} dm_1; \text{if } K_A \neq K'_A \text{ then stop};$

$N_B := \text{new nonce};$

$m_2 := \{[N_A, N_B, \text{pk}(SK_B)]\}_{K_A}; \text{send } m_2;$

$m_3 := \text{receive}; (N'_A, N'_B) \stackrel{?}{:=} \text{dec}(SK_B, m_3);$

if $N_A \neq N'_A \vee N_B \neq N'_B$ **then stop**;

SK_B is the decryption key of party B.

The whole protocol is

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$$\begin{aligned} & SK_A := \text{new asymkey}; SK_B := \text{new asymkey}; \\ & \text{send pk}(SK_A); \text{send pk}(SK_B); \\ & ( \\ & \quad !(K_X \stackrel{?}{:=} \text{receive}; P_A(K_X)) \mid \\ & \quad !(K_X \stackrel{?}{:=} \text{receive}; P_B(K_X)) \\ & ) \end{aligned}$$

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... and this is executed in parallel with the adversary.

- At each point of the process, only variables defined *above* this point are visible.
- We demand that no variable is ever redefined
 - no communication through shared variables.

Security properties:

- Secrecy of something — this thing cannot become the value of some variable in the adversarial process.
 - Generally a weaker property than “the adversary cannot distinguish which one of two fixed values this thing has”.
 - Justified by the perfection of the cryptographic primitives.
- Authenticity — a certain situation cannot happen...
 - B **thinks** it shares K_{AB} with A , but A **thinks** that K_{AB} is for a different purpose...

$P_A(K_B)$ is

$N_A :=$ new nonce; $K_{AB} :=$ new symkey;

. o O (start session with K_B using (N_A, K_{AB}))

$m_1 := \{\text{pk}(SK_A), N_A, K_{AB}\}_{K_B}$; send m_1 ;

$m_2 :=$ receive;

$dm_2 \stackrel{?}{:=} \text{dec}(SK_A, m_2)$;

$(N'_A, N_B, K'_B) \stackrel{?}{:=} dm_2$;

if $N_A \neq N'_A \vee K_B \neq K'_B$ then stop;

$m_3 := \{N_A, N_B\}_{K_B}$;

. o O (end session with K_B using (N_A, N_B, K_{AB}))

send m_3

$P_B(K_A)$ is

$m_1 := \text{receive}; dm_1 \stackrel{?}{:=} \text{dec}(SK_B, m_1);$

$(K'_A, N_A, K_{AB}) \stackrel{?}{:=} dm_1; \text{if } K_A \neq K'_A \text{ then stop};$

$N_B := \text{new nonce};$

. o O (start session with K_A using (N_A, N_B, K_{AB}))

$m_2 := \{[N_A, N_B, \text{pk}(SK_B)]\}_{K_A}; \text{send } m_2;$

$m_3 := \text{receive}; (N'_A, N'_B) \stackrel{?}{:=} \text{dec}(SK_B, m_3);$

if $N_A \neq N'_A \vee N_B \neq N'_B$ **then stop**;

. o O (end session with K_A using (N_A, N_B, K_{AB}))

Authentication property:

If B ended session with $\text{pk}(SK_A)$ using (n_1, n_2, k) then A ended session with $\text{pk}(SK_B)$ using (n_1, n_2, k) .

If A ended session with $\text{pk}(SK_B)$ using (n_1, n_2, k) then B started session with $\text{pk}(SK_A)$ using (n_1, n_2, k) .

... and for different red thoughts correspond different green thoughts.

Scheduling of protocols — **non-deterministic**.

We get a **set** of protocol traces, not a probability distribution over them.

Justification — both secrecy and authentication properties are specified by valid protocol traces.

In our actual arguments we just assume that everything that may go wrong goes wrong.

(A1) B ended session i with $K_A[i]$.

(A2) $K_A[i] = \text{pk}(SK_A)$.

(1) $m_3[i]$, which came from outside, contained the value of $N_B[i]$.

(2) $N_B[i]$ left the scope of the current session only inside $m_2[i]$.

(3) $m_2[i]$ was encrypted with $K_A[i] = \text{pk}(SK_A)$. Only someone who knows SK_A is able to decrypt it.

(4) SK_A is used only to get the corresponding public key, and to decrypt. Hence the adversary cannot know SK_A .

(5) A had a session j where she decrypted $m_2[i] = m_2[j]$.

Hence

– $N'_A[j] = N_A[i]$, $N_B[j] = N_B[i]$, $K'_B[j] = \text{pk}(SK_B)$.

– Maybe there were several such sessions j .

(6) $N_B[j]$ left the scope of the session j only inside $m_3[j]$.

– $K_B[j] = K'_B[j] = \text{pk}(SK_B)$, $N_A[j] = N'_A[j] = N_A[i]$.

– A ended session j with $K_B[j]$.

• We still have to show that

– $K_{AB}[j] = K_{AB}[i]$

– There is no $i' \neq i$, such that B ended session i' with $\text{pk}(SK_A)$ using $(N_A[i], N_B[i], K_{AB}[i])$.

* Easy — $N_B[i'] \neq N_B[i]$.

(7) $K_{AB}[i]$ is defined together with $N_A[i]$ which equals $N_A[j]$.

Can the adversary construct a message of the form

$\{\text{pk}(SK_A), N_A[i], K'\}_{\text{pk}(SK_B)}$ with $K' \neq K_{AB}[j]$?

(8) $N_A[j]$ is sent out in messages $m_1[j]$ and $m_3[j]$. They are encrypted with $\text{pk}(SK_B)$.

(9) The adversary does not know SK_B .

(10) B does not accept the message $m_3[j]$ as the first message from A.

(11) If B accepts $m_1[j]$ in some session k , then $K_A[k] = \text{pk}(SK_A)$. Hence the adversary cannot decrypt $m_2[k]$.

The adversary cannot learn $N_A[i]$.

The adversary cannot learn $N_A[i] = N_A[j]$ and there is only a single first message containing it constructed by A.

This message contains the key $K_{AB}[j]$.

Injective agreement would still hold if A's belief about ending a session had not contained N_B .

The other property is proved similarly.

Secrecy of K_{AB} is shown similarly to the secrecy of N_A .

Key-establishment protocols are just one case where authentication is necessary.

In pure authentication protocols ([entity authentication](#)) two parties have established a connection. Party A wants to check that the other one is who A thinks it is.

- In a connectionless model of communication, entity authentication is used to check the liveness of the other party.

Mutual authentication — both parties check each other's liveness.

Basic tool for one-way entity authentication: challenge-response mechanism.

- A sends a new nonce to B .
- B transforms that nonce in a way that only B (or A) could do and sends back the result.
- A checks the result.

Let $Cert_X$ be the certificate of the verification key $pk(K_X)$ of the party X .

Alice checking Bob's liveness:

$$A \longrightarrow B : N_A$$

$$B \longrightarrow A : Cert_B, N_A, N_B, A, \left[\{ N_A, N_B, A \} \right]_{pk(K_B)}$$

N_B is used to not let Alice completely control what is signed by Bob (otherwise K_B cannot be used for anything else).

(ISO Public Key Two-Pass Unilateral Authentication Protocol)

Mutual authentication — two unilateral authentications:

1. $A \longrightarrow B : N_{A1}$
2. $B \longrightarrow A : Cert_B, N_{A1}, N_B, A, \{ \{ N_{A1}, N_B, A \} \}_{pk(K_B)}$
3. $A \longrightarrow B : Cert_A, N_B, N_{A2}, B, \{ \{ N_B, N_{A2}, B \} \}_{pk(K_A)}$

A draft version of ISO Public Key Three-Pass Mutual Authentication Protocol.

- Simply two instances of the protocol on previous slide.
- Insecure.

1. $C(A) \longrightarrow B \quad : N_{A1}$
2. $B \longrightarrow C(A) : Cert_B, N_{A1}, N_B, A, [\{N_{A1}, N_B, A\}]_{pk(K_B)}$
- 1'. $C(B) \longrightarrow A \quad : N_B$
- 2'. $A \longrightarrow C(B) : Cert_A, N_B, N_{A2}, B, [\{N_B, N_{A2}, B\}]_{pk(K_A)}$
3. $C(A) \longrightarrow B \quad : Cert_A, N_B, N_{A2}, B, [\{N_B, N_{A2}, B\}]_{pk(K_A)}$

B thinks he has been the responder in a protocol session with A . A does not think that she has initiated a session with B .

A variant with no such attacks:

1. $A \longrightarrow B : N_A$
2. $B \longrightarrow A : Cert_B, N_A, N_B, A, \{ \{ N_A, N_B, A \} \}_{pk(K_B)}$
3. $A \longrightarrow B : Cert_A, N_B, N_A, B, \{ \{ N_B, N_A, B \} \}_{pk(K_A)}$

But here B has a lot of control over the message signed by A .

Exercise. What if A and B were not under signature in messages 2 and 3?

1. $A \longrightarrow C \quad : N_A$
- 1'. $C(A) \longrightarrow B \quad : N_A$
- 2'. $B \longrightarrow C(A) : Cert_B, N_A, N_B, A, [\{N_A, N_B\}]_{pk(K_B)}$
2. $C \longrightarrow A \quad : Cert_C, N_A, N_B, A, [\{N_A, N_B\}]_{pk(K_C)}$
3. $A \longrightarrow C \quad : Cert_A, N_B, N_A, C, [\{N_B, N_A\}]_{pk(K_A)}$
- 3'. $C(A) \longrightarrow B \quad : Cert_A, N_B, N_A, B, [\{N_B, N_A\}]_{pk(K_A)}$

B thinks he was the responder in a session initiated by A .

A does not think she had initiated a session with B .

Entity authentication can be done using one-time passwords:

A and B have agreed on a code-book $f : \{0, 1\}^n \longrightarrow \{0, 1\}^*$.

1. A generates $r \in \{0, 1\}^n$, sends it to B .
2. B responds with $f(r)$.
3. A checks that it indeed received $f(r)$.

Care has to be taken to not repeat the challenge r .

Lamport's one-time password scheme:

Initialization: B chooses a password pw and $n \in \mathbb{N}$. Sends $(B, h^n(pw), n)$ to A over an authenticated channel.

- B puts $n_B := n$.
- A puts $pw' := h^n(pw)$.

One round:

1. A sends a notice to B .
2. B computes $r := h^{n_B-1}(pw)$, decrements n_B and sends r to A .
3. A checks that $h(r) = pw'$ and puts $pw' := r$.

This works as long as A and B are synchronized. Resynchronization again requires authentic channels.

S/KEY one-time password scheme:

Initialization: B chooses a password pw and $n \in \mathbb{N}$. Sends $(B, h^n(pw), n)$ to A over an authenticated channel.

- A puts $n_A := n$.
- A puts $pw' := h^n(pw)$.

One round:

1. A sends the notice $n := n_A$ to B .
2. B computes $r := h^{n-1}(pw)$ and sends r to A .
3. A checks that $h(r) = pw'$, puts $pw' := r$ and $n_A := n - 1$.

Insecure. **Exercise.** Attack it.

We have seen Diffie-Hellman key exchange:

Let G be a group with hard Diffie-Hellman problem. Let g generate G . Let $m = |G|$.

1. A chooses a random $a \in \mathbb{Z}_m$, sends $x = g^a$ to B .
2. B chooses a random $b \in \mathbb{Z}_m$, sends $y = g^b$ to A .
3. A computes $K = y^a$. B computes $K = x^b$.
4. K is used as a common secret. ($h(K)$ may be a symmetric key)

This protocol needs authentication, too.

Station-to-station protocol:

$$A \longrightarrow B : g^{N_A}$$

$$B \longrightarrow A : g^{N_B}, Cert_B, \{ \{ \{ g^{N_B}, g^{N_A} \} \}_{K_B} \}_{g^{N_A N_B}}$$

$$A \longrightarrow B : Cert_A, \{ \{ \{ g^{N_A}, g^{N_B} \} \}_{K_A} \}_{g^{N_A N_B}}$$

Proposed by Diffie et al.

Aimed to have several security properties:

- Mutual entity authentication.
- Key agreement.
 - No third party knows the key.
- Key confirmation.
 - The other party knows the key.
- Perfect forward secrecy.

It does not quite achieve mutual authentication:

1. $A \longrightarrow C(B) : g^{N_A}$
- 1'. $C \longrightarrow B : g^{N_A}$
- 2'. $B \longrightarrow C : g^{N_B}, Cert_B, \{ \{ \{ g^{N_B}, g^{N_A} \} \}_{K_B} \}_{g^{N_A N_B}}$
2. $C(B) \longrightarrow A : g^{N_B}, Cert_B, \{ \{ \{ g^{N_B}, g^{N_A} \} \}_{K_B} \}_{g^{N_A N_B}}$
3. $A \longrightarrow C(B) : Cert_A, \{ \{ \{ g^{N_A}, g^{N_B} \} \}_{K_A} \}_{g^{N_A N_B}}$

At this point A thinks she was the initiator in a session with B . But B does not think he was a responder in a session with A .

The secrecy of $g^{N_A N_B}$ is not violated.

Identities of parties inside the signed messages would have helped.

Neumann-Stubblebine key exchange protocol.

A TTP T generates a new key for A and B .

Let K_{XT} be the (long-term) symmetric key shared by X and T .

1. $A \longrightarrow B : A, N_A$
2. $B \longrightarrow T : B, N_B, \{A, N_A, T_B\}_{K_{BT}}$
3. $T \longrightarrow A : N_B, \{B, N_A, K_{AB}, T_B\}_{K_{AT}}, \{A, K_{AB}, T_B\}_{K_{BT}}$
4. $A \longrightarrow B : \{A, K_{AB}, T_B\}_{K_{BT}}, \{N_B\}_{K_{AB}}$

T_B is a timestamp.

A similarity:

$$1. A \longrightarrow B : A, N_A$$

$$2. B \longrightarrow T : B, N_B, \{A, N_A, T_B\}_{K_{BT}}$$

$$3. T \longrightarrow A : N_B, \{B, N_A, K_{AB}, T_B\}_{K_{AT}}, \{A, K_{AB}, T_B\}_{K_{BT}}$$

$$4. A \longrightarrow B : \{A, K_{AB}, T_B\}_{K_{BT}}, \{N_B\}_{K_{AB}}$$

Attack through a type flaw:

1. $C(A) \longrightarrow B \quad : A, N_A$
2. $B \longrightarrow C(T) : B, N_B, \{A, N_A, T_B\}_{K_{BT}}$
4. $C(A) \longrightarrow B \quad : \{A, N_A, T_B\}_{K_{BT}}, \{N_B\}_{N_A}$

where $N_A \in \mathbf{Keys}_{\text{sym}} \cap \mathbf{Nonce}$.

B thinks he has agreed on key K_A with A . A has no idea.

Otway-Rees key exchange protocol:

1. $A \longrightarrow B : N, A, B, \{N_A, N, A, B\}_{K_{AT}}$
2. $B \longrightarrow T : N, A, B, \{N_A, N, A, B\}_{K_{AT}}, \{N_B, N, A, B\}_{K_{BT}}$
3. $T \longrightarrow B : \{N_A, K_{AB}\}_{K_{AT}}, \{N_B, K_{AB}\}_{K_{BT}}$
4. $B \longrightarrow A : \{N_A, K_{AB}\}_{K_{AT}}$

Possible type confusion:

1. $A \longrightarrow B : N, A, B, \{N_A, N, A, B\}_{K_{AT}}$
2. $B \longrightarrow T : N, A, B, \{N_A, N, A, B\}_{K_{AT}}, \{N_B, N, A, B\}_{K_{BT}}$
3. $T \longrightarrow B : \{N_A, K_{AB}\}_{K_{AT}}, \{N_B, K_{AB}\}_{K_{BT}}$
4. $B \longrightarrow A : \{N_A, K_{AB}\}_{K_{AT}}$

The triple (N, A, B) masquerading as a key may be from some old session.

Further reading:

Chapter 12.1–12.6 and 12.9 of

Menezes, van Oorschot, Vanstone.

Handbook of Applied Cryptography.

(available on-line)